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INSTITUTE OF TECHNOLOGY

AIR UNIVERSITY

UNITED STATES AIR FORCE

DTIC
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SCHOOL OF ENGINEERING

WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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AFIT/GA/AA/78D-3

Determination of a 12-Hour Periodic Orbit at the Critical Inclination with the Geopotential Represented through the Second Order Harmonics

Thesis

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Thesis

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air Training Command

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

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January 1979

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Preface

Several thanks are in order to those that assisted me in this study. First, to Capt William E. Wiesel who served as my thesis advisor. He showed remarkable understanding, and I am most grateful for the extra "push" he provided towards the end. I think we were both disappointed with the way things turned out, but due largely to Capt Wiesel's efforts we were able to make a statement that could be of value to other students.

Thanks also to Capt Gary Reid and Professor Robert Calico for their assistance in the evaluation of my thesis.

Finally, thanks to Cindy Held for her help in typing this paper.

Jim Garcia

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Abstract

The subject of this thesis was an attempt to find a periodic solution to the equations of motion of a high eccentricity (.7) satellite at the critical inclination, with period equal to 12 sidereal hours, and with the earth's potential represented through the second order harmonics. The goals of the study were to (1) find the periodic orbit by numerical means, (2) examine the stability of its motion, and (3) determine the characteristics of the motion near the periodic orbit after including the influences of the sun and moon in the equations of motion.

Success was obtained in the two-body case for all orbits and in the two-body + J_2 case for a circular equitorial orbit only. In the two-body + J_2 case, no periodic orbit could be found with non zero inclination and/or eccentricity $\neq 0$. No periodic orbits could be found when S_{22} and C_{22} were included in the equations of motion.

The equations of motion of the orbital elements were examined and used to explain why the periodic orbit could not be found.

Determination of a 12-Hour Periodic
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I Introduction

The subject of this thesis is an attempt to find a periodic solution to the equations of motion of a high eccentricity (.7) satellite at the critical inclination with period equal to 12 sidereal hours and with the earth's potential represented through the second order harmonics. The orbit is shown in Figure (1). Apogee is in the northern hemisphere with argument of perigee, ω, equal to 270°. The characteristics just described are very similar to those of the Soviet molynia communications satellite and a solution would be directly applicable to their motion. A solution would also be of general interest because of the special characteristics of such an orbit. The 12 sidereal hour period and the critical inclination both correspond to special cases in orbit theory.

The equations of motion of a satellite about the earth may be solved in two ways; either by using a general perturbations method to obtain an analytic expression for the motion or by numerical integration to obtain the solution over some particular time interval. The former approach is very convenient because the solution can be used for any orbit after evaluating the initial conditions for that orbit. However, general perturbations solutions become invalid when resonances exist between two or more frequencies of the motion. This difficulty is known as "the problem of small divisors" (Ref 6:128-133).

The critical inclination and 12 sidereal hour orbits are special cases because they correspond to resonance conditions.

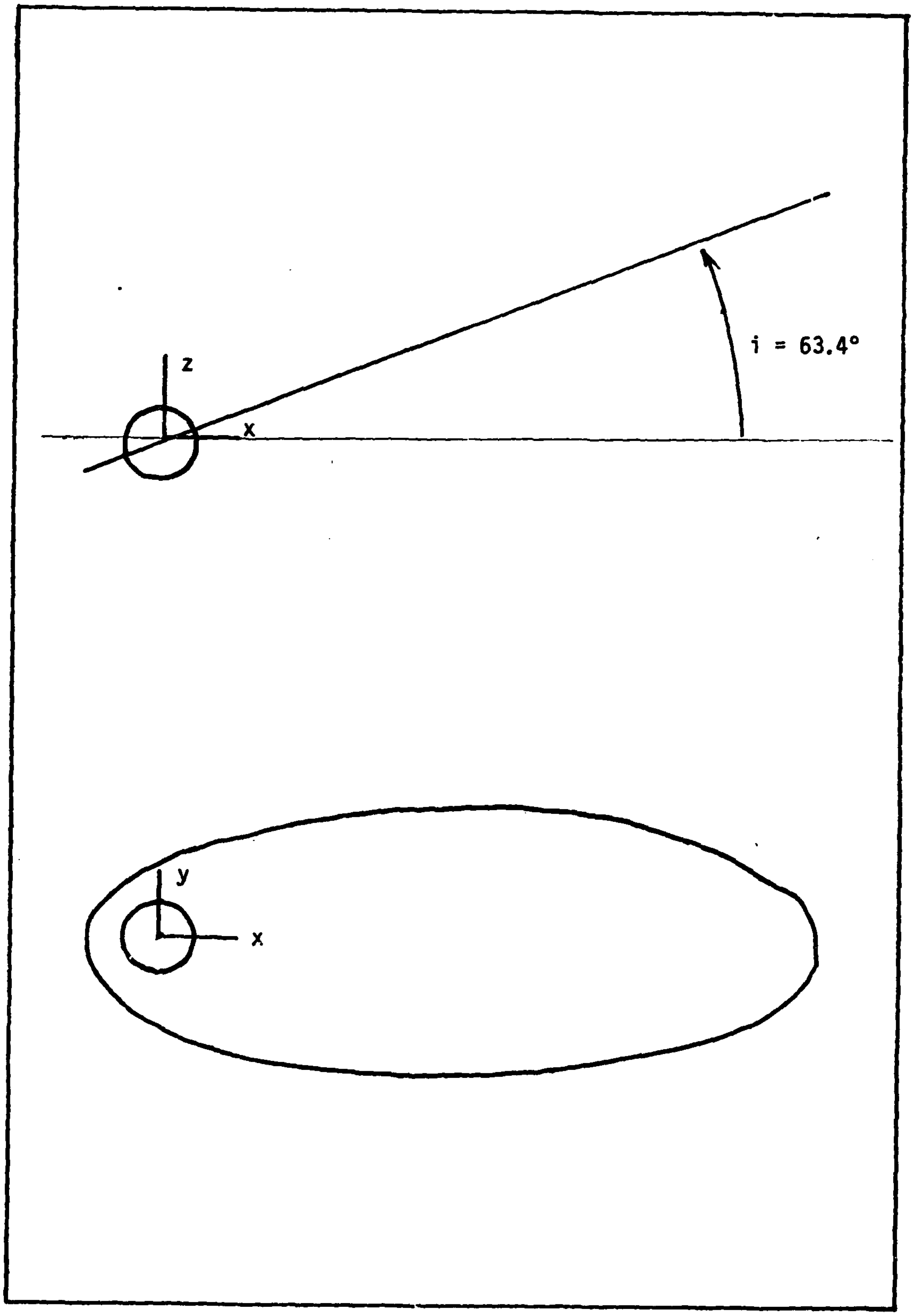

Figure 1. 12-Hour Periodic Orbit in Inertial Frame

Numerical integration of the equations of motion is very straightforward and does not suffer from resonance problems. However, the solution is good only for one orbit and over the time interval through which it is generated. As the interval of the integration becomes large, the amount of labor required and therefore the cost of generating the solution become prohibative. This problem can be solved if a periodic solution can be found. It has to be generated only once since any motion outside the solution interval is simply a repeat of that inside.

Hori's results (Ref 7) seemed to indicate that a periodic orbit did exist The original goals of this study became (1) find the periodic orbit by numerical means, (2) examine the stability of its motion, and (3) determine the characteristics of the motion near the periodic orbit after including the influences of the sun and moon in the equations of motion.

II Theory

Equations of Motion

The equations of motion of the satellite are determined relative to the coordinate system shown in Figure (2). The xyz-frame is fixed to the earth which is rotating with respect to inertial space with constant angular velocity $\Omega\hat{k}$. ϕ is the angle between the x-axis and the projection of the satellite's position vector in the x-y (equitorial) plane. The xyz-frame was chosen fixed to the earth so that later the Hamiltonian of the motion would be constant. The $\hat{e}$-frame is oriented such that $\hat{e}_r$ is pointed along the outward radial, $\hat{e}_\phi = \hat{k} x \hat{e}_r$, and $\hat{e}_\theta = \hat{e}_\phi x \hat{e}_r$. This set of coordinate axes becomes undefined when $\hat{e}_r \cdot \hat{k} = \pm 1$ so that the equations of motion developed using these coordinate axes cannot be used for polar orbits. The position of the satellite is

$$\bar{R} = r\hat{e}_r \tag{1}$$

where r is the distance between the earth's center of mass and the satellite The velocity of the satellite with respect to inertial space is

$$\bar{V} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + r(\dot{\phi}+\Omega)\sin\theta\hat{e}_\phi \tag{2}$$

The kinetic energy of the satellite is

$$T = \frac{1}{2}m\,\bar{V}\cdot\bar{V}$$

$$= \frac{1}{2}m\,(\dot{r}^2 + r^2\dot{\theta}^2 + r^2(\dot{\phi}+\Omega)^2\sin^2\theta) \tag{3}$$

The potential energy of the satellite is given by the expression (Ref 2:275, Ref 4:143)

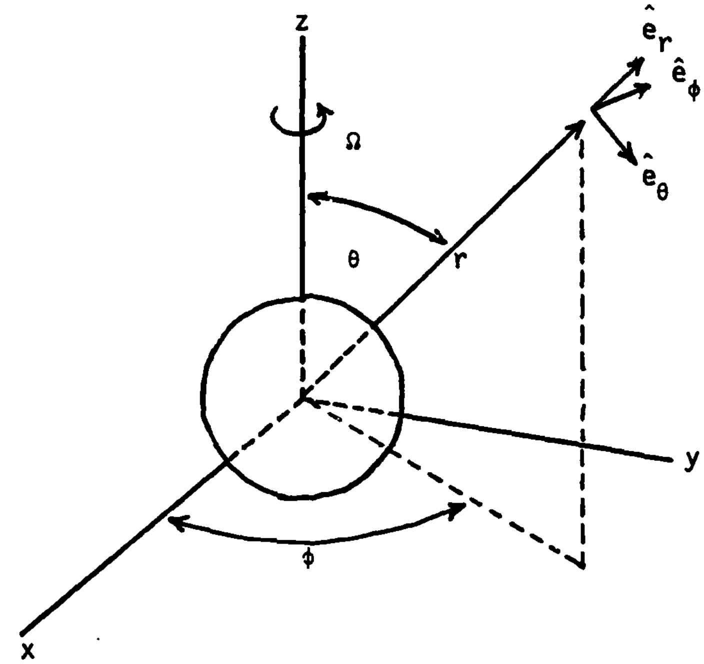

Figure 2. Spherical Coordinate Frame

$$V(r,\theta,\phi) = -\frac{GMm}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{R_e}{r}\right)^n P_n^m(\cos\theta)$$

$$[S_{nm} \sin m\phi + C_{nm} \cos m\phi] \quad (4)$$

where

G = universal gravitational constant

M = mass of earth

R_e = radius of earth

$P_n^m(\cos\theta)$ = associated Legendre polynomials

C_{nm}, S_{nm} = Geopotential coefficients

The terms with $m = 0$ are called zonal harmonics and depend on latitude only. The coefficients Cno are usually written J_n. The Sno coefficients are unimportant since $\sin(0\phi) = 0$. Terms with $n \neq m$ are called tesseral harmonics. Sectorals are those terms with $n = m$. Tesserals and sectorals are longitude dependent and satellites that make an integral number of revolutions about the earth in one sidereal day are in resonance with one order of the longitude dependent terms of the geopotential. A twelve hour satellite is in resonance with the $n \geq 2$, $m = 2$ terms. An eight hour satellite is in resonance with the $n \geq 3$, $m = 2$ terms, and so on. Because the 12-hour satellite is in longidutinal resonance, the C_{22}, S_{22} coefficients need to be retained in the potential.

If the origin is chosen to coincide with the earth's center of mass terms with $n = 1$ are zero (Ref 2:285), the J_2 coefficient is the most important with a magnitude on the order of 1×10^{-3} compared to 1×10^{-6} or smaller for the other coefficients. C_{21} and S_{21} are negligible (Ref 8). $P_0^0(\cos\theta) = 1$ so if C_{00} is set equal to 1, then the first term of the

potential is recognized as the two-body potential. Finally, the potential is through second order

$$V = -\frac{GMm}{r} - \frac{GMm\,R_e^{\,2}}{r^3} J_2 \frac{1}{2}(3\cos^2\theta - 1)$$

$$- \frac{GMm}{r^3} R_e^{\,2}\, 3\sin^2\theta\, (S_{22}\sin 2\phi + C_{22}\cos 2\phi) \qquad (5)$$

The lagrangian of the systems is given by:

$$L = T - V$$

$$= \frac{1}{2}m\,(\dot{r}^2 + r^2\dot{\theta}^2 + r^2(\dot{\phi}+\Omega)^2\sin^2\theta) + \frac{\mu m}{r}$$

$$+ \frac{\mu m R_e^{\,2}}{r^3} J_2 \frac{1}{2}(3\cos^2\theta - 1)$$

$$+ \frac{\mu m R_e^{\,2}}{r^3} 3\sin^2\theta(S_{22}\sin 2\phi + C_{22}\cos 2\phi) \qquad (6)$$

where $\mu(=GM)$ is the gravitational parameter. The Hamiltonian of the systems is given by $H = \Sigma p_i \dot{q}_i - L$ where q_i are the generalized coordinates and $p_i(=\partial L/\partial \dot{q}_i)$ are their conjugate momenta. The momenta and resulting Hamiltonian are

$$p_r = m\dot{r} \qquad (7)$$

$$p_\theta = mr^2\dot{\theta} \qquad (8)$$

$$p_\phi = mr^2(\dot{\phi}+\Omega)\sin^2\partial \qquad (9)$$

$$H = \frac{1}{2}m\,[p_r{}^2 + \frac{p_\theta{}^2}{r^2} + \frac{p_\phi{}^2}{r^2\sin^2\theta} - 2mp\phi\Omega] - \frac{\mu m}{r}$$

$$- \frac{\mu m R_e{}^2}{r^3} J_2 \frac{1}{2} (3\cos^2\theta - 1)$$

$$- \frac{\mu m R_e{}^2}{r^3} 3\sin^2\theta\ (S_{22}\sin 2\phi + C_{22}\cos 2\phi) \tag{10}$$

Since time, t, is not present in the right hand side of Eq (10), the Hamiltonian or energy of the system is constant. For two body motion and motion with J_2 not equal to zero, the $S_{22} = C_{22} = 0$ means that ϕ also does not appear in Eq (10). This means for those two cases p_ϕ is constant along with the Hamilotnian. The equations of motion are

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \quad , \quad \dot{p}_i = -\frac{\partial H}{\partial q_i} \tag{11}$$

which yield

$$\dot{y}_1 = y_4 \tag{12}$$

$$\dot{y}_2 = \frac{y_5}{y_1{}^2} \tag{13}$$

$$\dot{y}_3 = \frac{y_6}{y_1{}^2\sin^2 y_2} - \Omega \tag{14}$$

$$\dot{y}_4 = \frac{y_5{}^2}{y_1{}^3} + \frac{y_6{}^2}{y_1{}^3\sin^2 y_2} - \frac{1}{y_1{}^2} - \frac{3}{2} J_2 \frac{1}{y_1{}^4}(3\cos^2 y_2 - 1)$$

$$- \frac{9}{y_1{}^4} \sin^2 y_2 (S_{22}\sin 2y_3 + C_{22}\cos 2y_3) \tag{15}$$

$$\dot{y}_5 = \frac{y_6{}^2 \cos y_2}{y_1{}^2 \sin^3 y_2} - \frac{3}{2} J_2 \frac{1}{y_1{}^3} \sin 2y_2$$

$$+ \frac{6}{y_1{}^3} \sin y_2 \cos y_2 \, (S_{22} \sin 2y_3 + C_{22} \cos 2y_3) \qquad (16)$$

$$\dot{y}_6 = \frac{6}{y_1{}^3} \sin^2 y_2 \, (S_{22} \cos 2y_3 - C_{22} \sin 2y_3) \qquad (17)$$

where

$y_1 = r \qquad y_4 = \frac{p_r}{m}$

$y_2 = \theta \qquad y_5 = \frac{p_\theta}{m}$

$y_3 = \phi \qquad y_6 = \frac{p_\phi}{m}$

Units of mass, time, and distance were chosen such that $R_e = 1$ and $\mu = 1$. Eqs (12) - (17) are six first order differential equations of motion for the satellite.

Resonant Orbits

All two body orbits are periodic with respect to inertial space. In the rotating frame only resonant orbits, those that make an integral number of revolutions about the earth per day, are periodic. These are called sub-sychronous orbits and repeat the same groundtrack every 24 sidereal hours. Although they have different inertial periods, each has the same period in the rotating frame, that is, 24 hours. Figure (3) shows representative resonant orbits with inertial periods of 8, 12, and 24 sidereal hours.

Symmetry

Figure (3) shows that for the 12 hour orbit, apogee occurs at two points 180° apart. At apogee $\dot{r} = 0$ so that the orbit intersects the x-axis

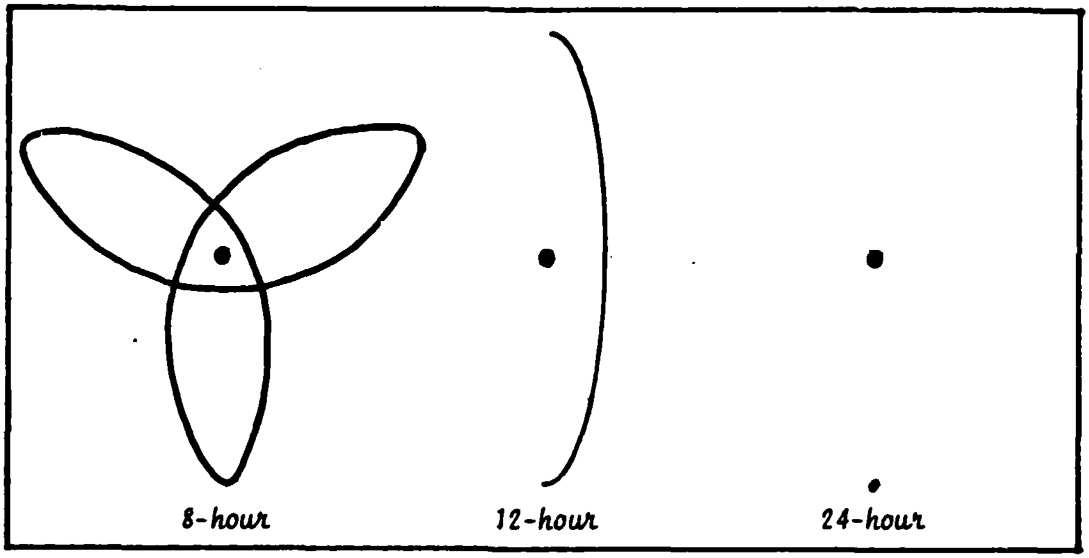

Figure 3. Resonant Orbits in Rotating Frame

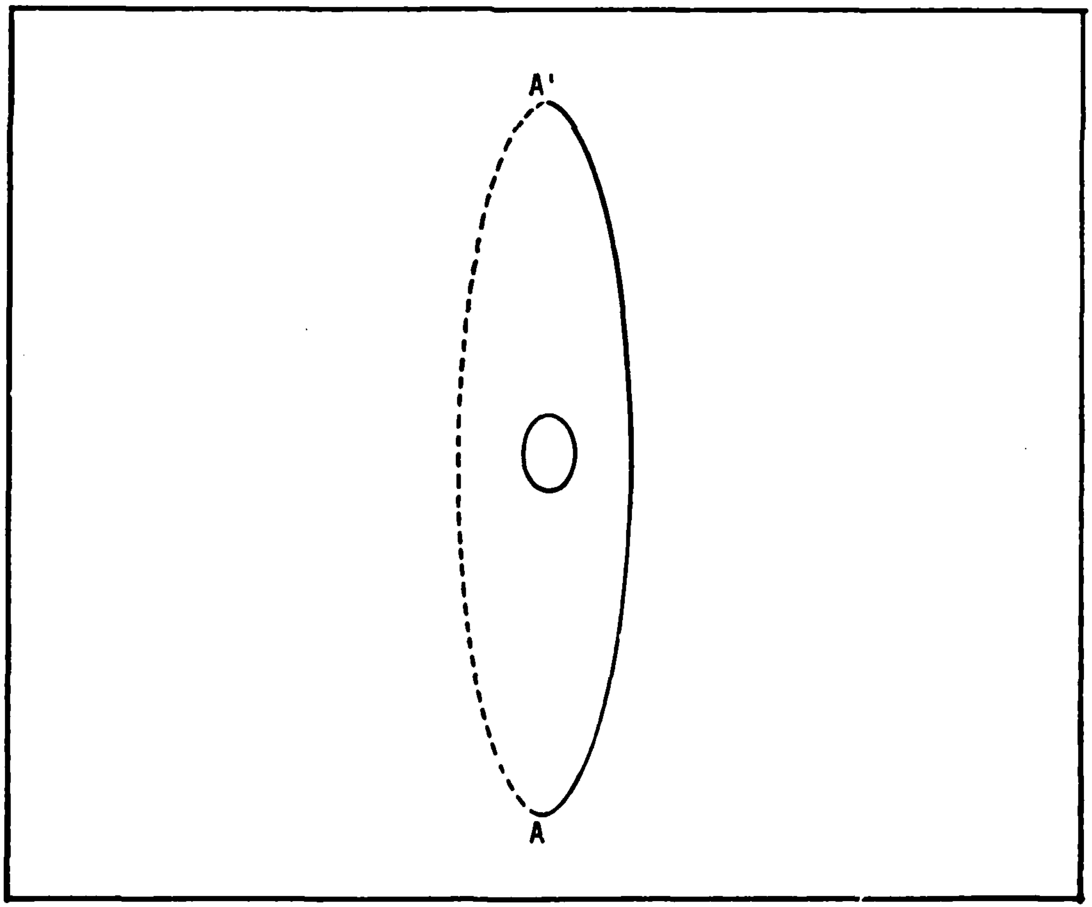

Figure 4. 12-Hour Orbit in Rotating Frame

perpendicularly. The path from A' to A (See Figure 4) must be identical to that from A to A' since they represent the same point in space. Notice the elliptical shape of the earth's equator. This represents the presence of the longitude dependent terms C_{22} and S_{22} in the geopotential and corresponds to the location of the Eurasian and American continents on the globe. By choosing apogee to line up along either the major or minor axis of the equitorial ellipse, the paths from A to A' and vice versa become mirror images of each other. The orbit is symmetric about the x-axis.

<u>Algorithm</u>

Consider the general equation of motion

$$\dot{\bar{x}} = \bar{f}(\bar{x}(t), t) \tag{18}$$

Suppose a solution $\bar{x}_0(t)$ is known. How does a nearby solution behave? If at the time t_0 the solution is displaced by a small amount $\delta\bar{x}(t_0)$, then the solution at a later time can be expanded as a Taylor series in $\delta\bar{x}(t_0)$, that is,

$$\bar{x}(t) = \bar{x}_0(t) + \left.\frac{\partial\bar{x}(t)}{\partial\bar{x}(t_0)}\right|_{x_0} \delta\bar{x}(t_0) + \text{Higher Order Terms} \tag{19}$$

if

$$\delta\bar{x}(t) = \bar{x}(t) - \bar{x}_0(t) \tag{20}$$

then

$$\delta\bar{x}(t) = \Phi(t,t_0)\delta\bar{x}(t_0) + \text{Higher Order Terms} \tag{21}$$

where

$$\Phi(t,t_0) = \frac{\partial\bar{x}(t)}{\partial\bar{x}(t_0)}$$

Φ is known as the state transition matrix and has the property $\Phi(t_0,t_0) = I$. To determing how $\Phi(t,t_0)$ propogates, recall from Eq (20)

$$\dot{\bar{x}}(t) = \bar{x}_0(t) + \delta\bar{x}(t) \tag{22}$$

using Eqs (22) and (18)

$$\dot{\bar{x}}(t) = \bar{f}(\bar{x}_0(t) + \delta\bar{x}(t),t)$$

$$= \bar{f}(\bar{x}_0(t),t) + \left.\frac{\partial\bar{f}}{\partial\bar{x}(t)}\right|_{x_0} \delta\bar{x}(t) + \text{Higher Order Terms} \tag{23}$$

or

$$\delta\dot{\bar{x}}(t) = A\,\delta\bar{x}(t) + \text{Higher Order Terms} \tag{24}$$

where

$$A = \frac{\partial\bar{f}(\bar{x}(t),t)}{\partial\bar{x}(t)} \tag{25}$$

substituting Eq (21) into Eq (24) and since $\delta\bar{x}(t_0)$ is arbitrary gives

$$\dot{\Phi}(t,t_0) = A(t)\Phi(t,t_0) \tag{26}$$

Eqs (18) and (26) can be integrated at the same time giving the solution $\bar{x}(t)$ and its local variation with initial conditions.

Suppose the period of the orbit is known (or chosen). One can guess the initial conditions, $\bar{x}_0(0)$, and integrate to get $\bar{x}_0(\tau)$. Since $\bar{x}_0(0)$ was a guess, the orbit probably will not close and some closure error will exist.

$$\Delta\bar{x} = \bar{x}(0) - \bar{x}(\tau) \quad . \tag{27}$$

Figure (5) shows the closure error. If the initial conditions are changed by an amount $\delta\bar{x}(0)$, then the solution at time τ will vary by an amount $\delta\bar{x}(\tau)$. The orbit will still not necessarily close unless $\Delta\bar{x} = \delta\bar{x}(\tau) - \delta\bar{x}(0)$. But

$$\delta\bar{x}(\tau) = \Phi(\tau,0)\delta\bar{x}(0)$$

so that for closure

$$\Delta\bar{x} = \Phi(\tau,0)\delta\bar{x}(0) - \delta\bar{x}(0)$$

$$= [\Phi(\tau,0) - I]\delta\bar{x}(0) \tag{28}$$

(See Figure (6)) Eq (28) represents n simultaneous equations in the n unknowns $\delta\bar{x}(0)$. Since higher order terms were neglected, the process should be iterated. The method is described by the following algorithm:

1) Guess initial conditions $\bar{x}_0(0)$

2) Integrate to get $\bar{x}_0(\tau)$ and $\Phi(\tau,0)$

3) Solve $[\Phi(\tau,0) - I]\delta\bar{x}(0) = \Delta\bar{x}_i$, where

   $$\Delta\bar{x}_i = \bar{x}_i(0) - \bar{x}_i(\tau)$$

4) $\bar{x}_{i+1}(0) = \bar{x}_i(0) + \delta\bar{x}(0)$

5) Are both $\delta\bar{x}(0)$ and $\Delta\bar{x}_i$ small enough?

   Yes → Stop
   No → Go to Step 2

<u>Singularity</u>. Solving Eq (28) for $\delta\bar{x}(0)$ requires the calculation of $[\Phi(\tau,0) - I]^{-1}$. For a periodic Hamiltonian system, $\Phi(\tau,0)$ will have 2 eigenvalues equal to 1 for each integral of the motion (Ref 9:430-433). In the two body and two body + J_2 cases there are two integrals of the motion, the Hamiltonian and p_ϕ. With C_{22} and S_{22} added, p_ϕ will no longer be constant but the Hamiltonian will remain so. This means that $[\Phi(\tau,0) - I]$ will be singular and so $[\Phi(\tau,0) - I]^{-1}$ is indeterminate.

To avoid the singularity problem, hold one or more of the initial

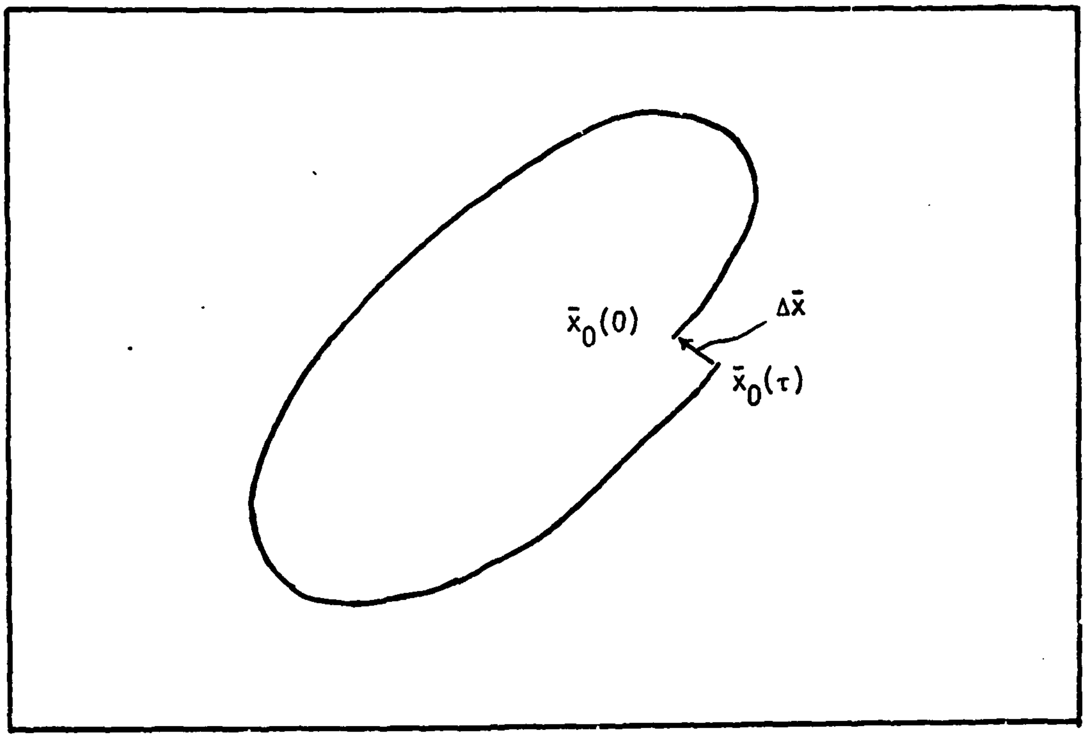

Figure 5. Closure Error of a Periodic Orbit (Ref 10)

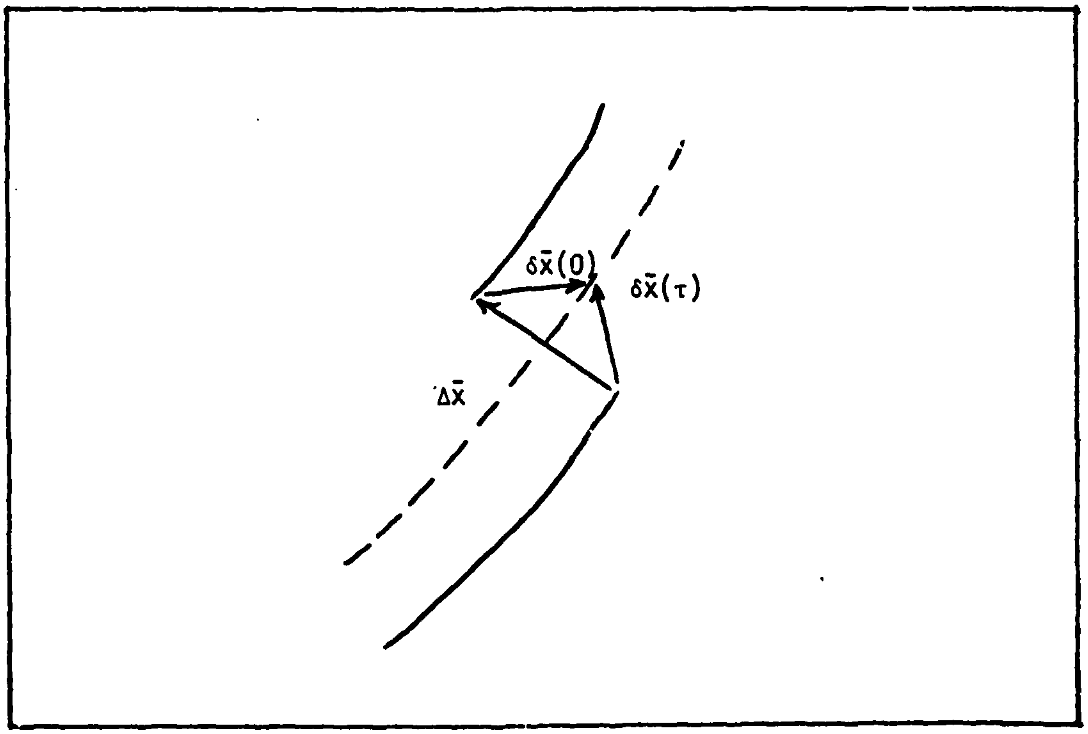

Figure 6. Initial Condition Changes of a Periodic Orbit (Ref 10)

conditions constant. The row and column of $[\Phi(\tau,0) - I]$ relating to the constant initial condition is then eliminated resulting in a matrix whose dimension is smaller than Φ and is not singular.

C_{22}, S_{22} motion has only one integral, the Hamiltonian. The magnitude of the 2nd order sectoral term is very small so that although p_ϕ is no longer constant, the change in p_ϕ is very small. This can result in the same singularity problem described above. While $[\Phi(\tau,0) - I]$ might not actually be singular, it can be so close to singular that the computer cannot distinguish any difference. The approach, then, is the same as before: Hold one of the initial conditions fixed.

<u>Modification</u>. By choosing the initial longitude of apogee equal to 0°, then three of the initial conditions become known, namely $\dot{r} = \dot{\theta} = \phi = 0$. That leaves three unknown initial conditions r, θ, and $\dot{\phi}$. At $\phi = 180°$, apogee occurs again and $\dot{r} = \dot{\theta} = 0$. The modified algorithm proceeds as follows:

1) Guess initial conditions r, θ, $\dot{\phi}$

   $\dot{r}$, $\dot{\theta}$, and ϕ are known

2) Integrate to get $\bar{x}_0(\tau)$ and $\Phi(\tau,0)$

   where $\bar{x}_0(\tau)$ and $\Phi(\tau,0)$ are defined as before

3) Solve $[\Phi'(\tau,0) - I]\delta\bar{x}_u(0) = \Delta\bar{x}_{k_i}(\tau)$

   where $\bar{x}_k$ is the vector of known initial conditions $\begin{matrix}\phi\\ \dot{r}\\ \dot{\theta}\end{matrix}$, $\bar{x}_u$ is the unknown initial conditions $\begin{matrix}r\\ \theta\\ \dot{\phi}\end{matrix}$, and $\Phi'(\tau,0)$ is the submatrix of $\Phi(\tau,0)$ defined by $\Phi'(\tau,0) = \dfrac{\partial\bar{x}_k(\tau)}{\partial\bar{x}_u(0)}$

4) $\bar{x}_{u_{i+1}}(0) = \bar{x}_{u_i}(0) + \delta\bar{x}_u(0)$

5) Evaluate the residuals $\Delta\bar{x}_i$ and $\delta\bar{x}_u(0)$

Are both small enough?

Yes → Stop

No → Go to Step 2

Program

The integration of Eqs (12) - (17) and (26) was carried out using a numerical integration package named ODE (Ordinary Differential Equations). This package is part of the CC6600 library for use on the CDC6600 computers at AFIT. The program is listed in Appendix B.

The elements of the matrix A were needed in the program.

$$a_{ij} = \frac{\partial f_i}{\partial x_j} \tag{29}$$

The partial derivatives of Eqs (12) - (17) were taken and elements of A determined by Eq (29). Appendix A lists the derivatives.

Verification of A and Φ Matrices

Recall Eqs (25), (26), and (29)

$$A = \frac{\partial \bar{f}}{\partial \bar{x}} \tag{25}$$

$$\dot{\Phi}(t,t_0) = A(t)\,\Phi(t,t_0) \tag{26}$$

$$a_{ij} = \frac{\partial f_i}{\partial x_j} \tag{29}$$

and that $\Phi(0,0) = I$. In order to verity that the partials $\partial f_i/\partial x_j$ were taken correctly and that Φ is being propogated correctly, alternate methods were used to form both A and Φ and compared to the results given by Eqs (25) and (26).

Elements of A. Given a solution $\bar{x}(t)$, then from Eqs (12) - (17) $\bar{f}(t)$ can be evaluated. Perturbing the jth element of $\bar{x}(t)$ by an amount Δx_j gives the solution $\bar{x}_p(t)$. $\bar{f}_p(t)$ can then be evaluated. The jth column of A is then approximately

$$A_{p_j} = \frac{\bar{f}_p - \bar{f}}{\Delta x_j} \tag{30}$$

Each column of A can be obtained in a similar manner. The results of Eq (30) can be compared to those of Eq (25) and checked for agreement.

Φ Propogation. *The proof that Φ is being propogated correctly is very* similar to the one just discussed for the A matrix. Given the initial conditions $\bar{x}_0(0)$ integrate to get $\bar{x}(t)$. Next perturb the jth element of $\bar{x}_0(0)$ to get $\bar{x}_p(0)$. Integrate again to get $\bar{x}_p(t)$. The jth column of Φ is approximately

$$\Phi_{p_j}(t,0) = \frac{\bar{x}_p(t) - \bar{x}(t)}{\Delta x_j(0)} \tag{31}$$

Each column of Φ_p can be generated in a similar manner. The results of Eq (31) and the integration of Eq (22) can be compared and checked for agreement.

III Results and Discussion

Results

The formation of the A matrix and propogation of the Φ matrix were verified using the methods described earlier. Tables (1) and (2) list the results of the comparisons showing good agreement in both cases. Very large values were used for J_2, S_{22}, and C_{22} so that any errors in the terms associated with those coefficients would be discernable.

Another check of the program consisted of using the equations of motion (Eqs (12) - (17)) to verify the location of four equilibrium points aligned along the principal axes of the equitorial eclipse (Ref 5:167-169). Setting the right hand sides of Eqs (12) - (17) to zero and with $\theta = 90°$ gave the four equilibrium points.

With $J_2 = S_{22} = C_{22} = 0$ the program was tested to see if two body resonant orbits converged. First, precalculated two body elements were inputted into the program and tested for immediate convergence. Convergence was obtained in one iteration in most instances and always within two iterations. Second, elements that were slightly displaced from the two body elements were inputted into the program. Convergence to the two body orbit was obtained very quickly. This was done for both circular and elliptical orbits, inclined to and in the plane of the equator. Last, to show that non-resonant orbits do not close in the rotating frame, an orbit was inputted into the program which then tried to force convergence to the non-resonant orbit. Convergence could not be obtained which was interpreted as proof that non-resonant orbits are not periodic in the rotating frame.

It can be easily seen that the modified algorithm has many variations depending on the choice of known and unknown initial conditions. For the two body problem, if $r_A(0)$ and τ are known, then $\dot{\phi}(0)$ can be calculated.

TABLE 1. RESULTS OF A MATRIX VERIFICATION

A MATRIX

0.	0.	0.	.100E+01	0.	0.
-.200E+01	0.	0.	0.	.100E+01	0.
-.282E+01	-.181E+01	0.	0.	0.	.141E+01
-.519E+01	-.177E+01	.795E-01	0.	.200E+01	.282E+01
-.177E+01	-.315E+01	0.	0.	0.	.181E+01
.795E-01	0.	-.197E-01	0.	0.	0.

PERTURBED A MATRIX

0.	0.	0.	.100E+01	0.	0.
-.200E+01	0.	0.	0.	.100E+01	0.
-.282E+01	-.181E+01	0.	0.	0.	.141E+01
-.519E+01	-.177E+01	.795E-01	0.	.200E+01	.282E+01
-.177E+01	-.315E+01	0.	0.	0.	.181E+01
.795E-01	0.	-.197E-01	0.	0.	0.

TABLE 2. RESULTS OF PHI MATRIX VERIFICATION

PHI MATRIX

.110E+01	-.625E+00	-.198E+00	.110E+01	.152E+00	.485E+00
-.729E-02	.103E+01	-.336E-01	.131E+00	.630E-01	-.306E-01
-.529E+00	.385E+01	.108E+01	-.274E+00	-.520E-01	-.325E+01
.136E+00	-.522E+00	-.149E-01	.165E+01	.992E-02	.632E+00
-.818E+00	.496E+01	.986E-01	-.364E+00	.929E+00	-.364E+01
.149E-01	-.191E+00	-.538E-01	.193E+00	.376E-01	.108E+01

PERTURBED PHI MATRIX

.106E+01	-.426E+00	-.194E+00	.338E+00	.139E+00	.327E+00
-.319E-02	.101E+01	-.342E-01	.133E+00	.413E-01	-.111E-02
-.529E+00	.392E+01	.107E+01	-.265E+00	-.435E-01	-.327E+01
.135E+00	-.316E+00	-.147E-01	.105E+01	.930E-02	.627E+00
-.813E+00	[illegible]	[illegible]	[illegible]	.933E+00	-.392E+01
.149E-01	-.191E+00	[illegible]	.193E+00	.355E-01	.108E+01

Check of a matrix had $R = \theta = \phi = P_r = P_\theta = P_\phi = 1$, and time equal zero. Check of PHI matrix had initial conditions $R = 7.0796$, $\theta = 26.6°$, $\phi = 22.5°$, $P_r = P_\theta = 0$, $P_\phi = .6525$, and was integrated to $t = 53.3979$. Both programs and $\Delta x_j = 1.E-5$ and $C_{22} = S_{22} = 1.E-2$, $J_2 = 1.083E-2$.

Since the integration begins at apogee $\dot{r}(0)$ and $\dot{\theta}(0)$ are known. $\theta(0)$ and $\phi(0)$ are arbitrary and so are also known. The algorithm is modified so that

$$\bar{x}_k(\tau) = \begin{matrix} r \\ \theta \\ \phi \\ \dot{r} \\ \dot{\theta} \end{matrix} \quad , \qquad \bar{x}_u(0) = \{p_\phi\}$$

next, J_2 was included in the equations of motion and the same choice of known and unknown initial conditions was made. Convergence was obtained for circular, equitorial orbits but periodic orbits could not be obtained that were inclined to the equator or elliptical. When C_{22} and S_{22} were included along with J_2 then convergence could not be obtained in any case.

Discussion

After repeated failures to find the periodic orbit, it became obvious that the algorithm being used was not working. Either the computer algorithm and/or equations of motion were incorrect or the periodic orbit being searched for did not exist. At first the former was thought to be the most likely, but the good agreement obtained in the check of the A and ϕ matrices along with the success of the two body orbit check indicated that the program was working correctly. If the periodic orbit does not exist , then the question becomes "why not?"

This study was begun on the basis of Hori's (Ref 7) results which were believed to indicate that the periodic orbit did exist. It is now believed that those results were misunderstood. General perturbation theory gives for the time derivatives of the orbital elements (Ref 3)

$$\dot{\Omega} = -\frac{3}{2}\left(\frac{R_e}{a}\right)^2 \frac{J_2 n}{(1-e^2)^2} \cos i \tag{32}$$

$$\dot{\omega} = -\frac{3}{4}\left(\frac{R_e}{a}\right)^2 \frac{J_2 n}{(1-e^2)^2} (1 - 5 \cos^2 i) \tag{33}$$

where

i = inclination

a = semi-major axis

Ω = longitude of the ascending node

ω = argument of perigee

e = eccentricity

n = mean motion

At the critical inclination $\dot{\omega} = 0$ (the critical inclination is known as such because it is the inclination at which the rotation of the apses changes direction). Eq (32) predicts the rotation of the node and could explain why the algorithm failed to find a periodic orbit. At first it was thought that since the node seemed to rotate by 180° each period that the very small rotation predicted by Eq (32) would be negligible. This may not be the case. The success obtained in the two body + J_2 case for the circular equitorial orbit could have disguised the nodal regression which cannot be seen in such an orbit. If nodal regression provides the clue as to failure occurred, then the supposed period of 12 hours may be incorrect. The correct period of the periodic orbit if it exists is probably related to the magnitude of J_2. In fact, Barrar's work (Ref 1) proves the existence of periodic orbits at the critical inclination about an oblate earth whose periods are on the order of

$$\frac{1}{J_2^{3/2}}$$

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Appendix A

Elements of A Matrix

$$a_{11} = a_{12} = a_{13} = a_{15} = a_{16} = 0 \tag{A-1}$$

$$a_{14} = 1 \tag{A-2}$$

$$a_{21} = -2 \frac{y_5}{y_1{}^3} \tag{A-3}$$

$$a_{22} = a_{23} = a_{24} = a_{26} = 0 \tag{A-4}$$

$$a_{25} = \frac{1}{y_1{}^2} \tag{A-5}$$

$$a_{31} = \frac{-2y_6}{y_1{}^2 \sin^3 y_2} - \Omega \tag{A-6}$$

$$a_{32} = \frac{-2y_6 \cos y_2}{y_1{}^2 \sin^3 y_2} \tag{A-7}$$

$$a_{33} = a_{34} = a_{35} = 0 \tag{A-8}$$

$$a_{36} = \frac{1}{y_1{}^2 \sin^2 y_2} \tag{A-9}$$

$$a_{41} = \frac{-3y_5{}^2}{y_1{}^4} - \frac{3y_6{}^2}{y_1{}^4 \sin^2 y_2} + \frac{2}{y_1{}^3} + \frac{6J_2}{y_1{}^5} (3\cos^2 y_2 - 1)$$

$$+ \frac{36}{y_1{}^5} \sin^2 y_2 \; (S_{22} \sin 2y_3 + C_{22} \cos 2y_3) \tag{A-10}$$

$$a_{42} = \frac{-2y_6^2 \cos y_2}{y_1^3 \sin^3 y_2} + \frac{9J_2}{y_1} \cos y_2 \sin y_2$$

$$- \frac{18}{y_1^4} \sin y_2 \cos y_2 \; (S_{22} \sin 2y_3 + C_{22} \cos 2y_3) \qquad \text{(A-11)}$$

$$a_{43} = - \frac{18}{y_1^4} \sin^2 y_2 \; (S_{22} \cos 2y_3 - C_{22} \sin 2y_3) \qquad \text{(A-12)}$$

$$a_{44} = 0 \qquad \text{(A-13)}$$

$$a_{45} = \frac{2y_5}{y_1^3} \qquad \text{(A-14)}$$

$$a_{46} = \frac{2y_6}{y_1^3 \sin^2 \; _2} \qquad \text{(A-15)}$$

$$a_{51} = \frac{-2y_6^2 \cos y_2}{y_1^3 \sin^3 y_2} + \frac{9}{2} \frac{J_2}{y_1^4} \sin 2y_2$$

$$- \frac{18}{y_1^4} \sin y_2 \cos y_2 \; (S_{22} \sin 2y_3 + C_{22} \cos 2y_3) \qquad \text{(A-16)}$$

$$a_{52} = \frac{-3y_6^2 \cos^2 y_2}{y_1^2 \sin^4 y_2} - \frac{y_6^2}{y_1^2 \sin y_2} - \frac{3J_2}{y_1^3} \cos 2y_2$$

$$+ \frac{6}{y_1^3} (\cos^2 y_2 - \sin^2 y_2) \; (S_{22} \sin 2y_3 + C_{22} \cos 2y_3) \qquad \text{(A-17)}$$

$$a_{53} = \frac{12}{y_1^3} \sin y_2 \cos y_2 \; (S_{22} \cos 2y_3 - C_{22} \sin 2y_3) \qquad \text{(A-18)}$$

$$a_{54} = a_{55} = 0 \tag{A-19}$$

$$a_{56} = \frac{2y_6 \cos y_2}{y_1{}^2 \sin^3 y_2} \tag{A-20}$$

$$a_{61} = -\frac{18}{y_1{}^4} \sin^2 y_2 \, (S_{22}\cos 2y_3 - C_{22}\sin 2y_3) \tag{A-21}$$

$$a_{62} = \frac{12}{y_1{}^3} \sin y_2 \cos y_2 \, (S_{22}\cos 2y_3 - C_{22}\sin 2y_3) \tag{A-22}$$

$$a_{63} = -\frac{12}{y_1{}^3} \sin^2 y_2 \, (S_{22}\sin 2y_3 + C_{22}\cos 2y_3) \tag{A-23}$$

$$a_{64} = a_{65} = a_{66} = 0 \tag{A-24}$$

APPENDIX B

Computer Program to Calculate Periodic Orbits

```
      PROGRAM ORBIT(INPUT=/80,OUTPUT,TAPE5=OUTPUT)
C
C     THIS PROGRAM WAS USED TO FIND PERIODIC ORBITS IN THE TWO-BODY+J2+
C     S22,C22 PROBLEM.  THE PROGRAM SEGMENTS ARE DESCRIBED BELOW.
C     THE SIX PHASE SPACE COORDINATES ARE R,THETA,PHI,P/R,P/THETA, AND
C     P/PHI.    , THETA, AND PHI ARE THE GENERALIZED COORDINATES WHILE P/R
C     ,P/THETA AND P/PHI ARE THEIR CONJUGATE MOMENTA.  THE ARRAY YK IS
C     THE VECTOR OF KNOWN INITIAL CONDITIONS, YU IS THE VECTOR OF UNKNOWN
C     INITIAL CONDITIONS, AND DELYU IS THE VECTOR CHANGE TO THE UNKNOWN
C     INITIAL CONDITIONS.  IN THIS EXAMPLE PROGRAM (ONE OF MANY POSSIBLE
C     VARIATIONS) THERE IS ONLY ONE UNKNOWN INITIAL CONDITION,P/PHI.
C     THE VECTOR YK IS THEN
C
C               YK=(R,THETA,PHI,P/R,P/THETA)
C     ALSO
C                     YU=(P/PHI)
C
      EXTERNAL F
      DIMENSION Y(42),YP(42),WORK(382),IWORK(5)
      DIMENSION DELYU(1),YU(1),YK(5)
      DIMENSION W(6),WK(6),PHI2(6,6),ALPHA(6)
      DIMENSION DIF(6,6)
      DIMENSION RES(6)
      COMMON/COM1/PHI(6,6),PHIDOT(6,6)
      COMMON/COM2/A(6,6)
      COMMON XM,XK1,XK2,XJ2,S22,C22,OME
      REAL IDT(6,6)
      COMPLEX W
      COMPLEX ALPHA
      REAL LAM
```

```
      COMMON/ZA/LA1
      REAL LNG
    6 FORMAT(1X,3E20.10)
    7 FORMAT(//1X,5HPHIRM)
 1000 FORMAT(1X,2E20.10)
 1001 FORMAT(1X,120(1H*))
 2000 FORMAT(1X,19HEIGENVALUES OF PHI/5X,9HREAL PART,9X,14HIMAGINARY PAR
     $T)
 2001 FORMAT(1X,26HCHARACTERISTIC EXPONENTS/5X,9HREAL PART,9X,14HIMAGINA
     $RY PART)
 2011 FORMAT(1X,15HITERATION NO.: ,I5)
 3000 FORMAT(1X,5HMASS: ,F5.3,9X,5HC22: ,E10.4,5X,5HNEQN: ,I2,12X,
     *7HIFLAG: ,I2/1X,4HK1: ,F5.3,11X,5HS22: ,E10.4,5X,8HRELERR: ,
     *E10.4,9X,11HF INITIAL: ,F5.3/1X,4HK2: ,F5.3,11X,4HJ2: ,E10.4,6X,
     *8HABSERR: ,E10. ,2X,6HTOUT: ,F7.3/1X,5HOME: ,E10.4/)
 3011 FORMAT(1X,4HRES(,I2,3H)= ,E15.10)
 4000 FORMAT(1X,5(2HY(,I2,3H)= ,F10.6,5X))
 5000 FORMAT(1X,3E20.10)
 6000 FORMAT(1X,5E20.10)
 7000 FORMAT(//1X,24HPHI (STATE TRANSITION MATRIX))
 8000 FORMAT(1X,5HDELYU(,I2,3H)= ,E16.10)
C
C     THE PROGRAM READS IN:
C
C     XM=MASS OF SATELLITE
C     XK1=GM, THE GRAVITATIONAL PARAMETER (MU)
C     XK2=GM(RE**2), RE=MEAN EARTH RADIUS
C     S22,C22=SECOND ORDER SECTORAL COEFFICIENTS
C     OME=OMEGA, ROTATION RATE OF THE EARTH
C
```

```
C     THE UNITS OF MASS, TIME, AND DISTANCE WERE CHOSEN SUCH THAT XK1
C     AND XK2=1(SEE PAGE    ).  WHEN THE EQUATIONS OF MOTION WERE FIRST
C     DERIVED THE PHASE SPACE COORDINATES WERE R,THETA,PHI,P/R,P/THETA,
C     AND P/PHI.  LATER THEY WERE TRANSFORMED TO BE R,THETA,PHI,
C     (P/R)/XM,(P/THETA)/XM, AND (P/PHI)/XM, WHERE XM IS THE MASS OF THE
C     SATELLITE.  THAT MEANT THAT XM DID NOT APPEAR ON THE RIGHT HAND
C     SIDES OF EQS(12)-(17)(PAGE    ).  WHEN THIS PROGRAM WAS WRITTEN:
C     HOWEVER, THE TRANSFORMATION OF COORDINATES HAD NOT YET BEEN MADE
C     THAT IS WHY XM APPEARS IN THE EQUATIONS HERE.  BY SETTING XM=1 WE
C     GET THE MODIFIED EQUATIONS.  TO AVOID CONFUSION, THE TERMS XM, XK1
C     AND XK2 SHOULD BE REMOVED FROM THE PROGRAM SINCE THEY ARE ALL EQUAL
C     TO 1 ANYWAY.
C
      READ*,XM,XK1,XK2,XJ2,S22,C22,OME
C
C     THE PARAMETERS NEQN,RELERR,ABSERR,IFLAG,T,TOUT ARE USED BY THE
C     INTEGRATION PACKAGE ODE.  T IS THE STARTING TIME OF THE INTEGRA-
C     TION AND FOR OUR APPLICATION EQUALS ZERO.  TOUT IS THE FINAL TIME
C     OF THE INTEGRATION AND EQUALS ONE PERIOD OR 12 SIDEREAL HOURS.
C
      READ*,NEQN,RELERR,ABSERR,IFLAG,T,TOUT
      PRINT(6,3000)XM,C22,NEON,IFLAG,XK1,S22,RELERR,T,XK2,XJ2,ABSERR
     *,TOUT,OME
C
C     THE TWO-BODY ORBIT IS USED AS AN INITIAL GUESS TO THE UNKNOWN
C     INITIAL CONDITIONS. THE ECCENTRICITY, INCLINATION, AND INITIAL
C     LONGITUDE ARE READ BY THE PROGRAM.  GIVEN THE PERIOD(TOUT) AND
C     THE ECCENTRICITY(EC), THE RADIUS OF APOGEE(RA) IS DETERMINED.
C     SUBROUTINE INITIAL USES THE GIVEN INFOMATION TO CALCULATE THE
C     TWO-BODY ORBIT CONDITIONS.  (ABOUT THE ORBIT: RECALL FROM PAGE
```

```
C     THAT APOGEE IS IN THE NORTHERN HEMISPHERE WITH ARGUEMENT OF PERIGEE
C     EQUAL 270 DEGREES.)
C
      READ*,EC,XIN,LNG
      READ*,LAM
      PAR=6.2831853072-OME*TOUT
      X=(TOUT/6.2831853071)**.6666666667
      RA=X/(1.+EC)*6378.145
      CALL INITIAL(RA,EC,XIN,LNG,Y,OME)
      PRINT 1011
      YM(1)=Y(5)
      YK(1)=Y(1)
      YK(2)=Y(2)
      YK(3)=Y(3)
      YK(4)=Y(4)
      YK(5)=Y(5)
      DO 15 J=1,6
      DO 15 K=1,6
      IDEN(J,K)=0.
      IF(J.EQ.K)IDEN(J,K)=1.
      L=6*J+K
   15 Y(L)=IDEN(J,K)
      PRINT(6,4100)(L,Y(L),L=1,42)
      PRINT 1011
      CALL F(T,Y,YP)
      CALL PRINT(T,Y)
      PRINT 1011
      ICOUNT=1
   20 C2=COS(Y(2))
      S2=SIN(Y(2))
```

```
      A22=SQRT(C22**2+S22**2)
      C3=COS(2.*Y(3)+LAM)
      S3=SIN(2.*Y(3)+LAM)
C
C     THE HAMILTONIAN BEFORE THE INTEGRATION IS CALCULATED TO BE COMPARED
C     LATER TO THE HAMILTONIAN AFTER THE INTEGRATION.  RECALL THAT THE
C     HAMILTONIAN SHOULD BE CONSTANT(SEE PAGE    ).
C
      HO=(Y(4)**2+(Y(5)/Y(1))**2+(Y(6)/(Y(1)*S2))**2)*.5/XM-XK1/Y(1)-
     *XK2*XJ2*(.25+.75*COS(2.*Y(2)))/Y(1)**3-XK2*A22*S3/Y(1)**3-Y(6)*OME
C
C     THE INTEGRATION IS CARRIED OUT BY ODE AND THE RESIDUALS ARE CALCU-
C     LATED.  THE VECTOR Y IS
C                  Y=(R,THETA,PHI,P/R',P/THETA',P/PHI')
C     THE (') INDICATES (/XM) AND WILL BE DROPPED FOR SIMPLICITY.
C     RES(3) IS THE RESIDUAL IN PHI,THE LONGITUDE.  IN ONE PERIOD PHI
C     CHANGES BY 180 DEGREES.  THEREFORE, THE PHI RESIDUAL IS GIVEN BY
C                 RES(3)=PHI(T=0)+180 DEGREES-PHI(T=TOUT)
C                       =YK(3)+3.14159265+-Y(3)
C
   40 CALL ODE(F,NEQN,Y,T,TOUT,RELERR,ABSERR,IFLAG,WORK,IWORK)
      IF(T.EQ.TOUT)GO TO 30
      GO TO 40
   30 RES(1)=YK(1)-Y(1)
      RES(2)=YK(2)-Y(2)
      RES(3)=YK(3)-Y(3)+3.141323131
      RES(4)=YK(4)-Y(4)
      RES(5)=YK(5)-Y(5)
      RES(6)=YU(1)-Y(6)
      S2=SIN(Y(2))
```

```
      C2=COS(Y(2))
      A22=SQRT(C22**2+S22**2)
      C3=COS(2.*Y(3)+LAM)
      S3=SIN(2.*Y(3)+LAM)
      HA=(Y(4)**2+(Y(5)/Y(1))**2+(Y(6)/(Y(1)*S2))**2)*.5/XM-XK1/Y(1)-
     *XK2*XJ2*(.25+.75*COS(2.*Y(2)))/Y(1)**3-XK2*A22*S3/Y(1)**3-Y(6)*OME
C
C     THE MATRIX (PHI-I) IS FORMED, WHERE HERE PHI STANDS FOR THE STATE
C     TRANSITION MATRIX.  THE EIGENVALUES OF PHI ARE DETERMINED ALONG
C     WITH THE CHARACTERISTIC EXPONENTS.  THIS INFORMATION WILL BE USE-
C     FUL IN THE STABILITY ANALYSIS AND ALSO AS A CHECK THAT A PERIODIC
C     ORBIT HAS BEEN OBTAINED(SINCE THERE WILL BE TWO EXPONENTS EQUAL TO
C     ZERO FOR EVERY CONSTANT OF THE MOTION FOR THE PERIODIC ORBIT).
C
      DO 65 I=1,6
      DO 65 J=1,6
      PHI2(I,J)=PHI(I,J)
   65 DIF(I,J)=PHI(I,J)-IDEN(I,J)
      IJOB=0
      N=6
      IA=6
      CALL EIGRF(PHI2,N,IA,IJOB,W,Z,IZ,WK,IER)
      DO 70 I=1,6
      WR=REAL(W(I))
      WI=AIMAG(W(I))
      AR=ALOG(SQRT(WR**2+WI**2))/TOUT
      AI=ATAN2(WI,WR)/TOUT
   70 ALPHA(I)=CMPLX(AR,AI)
C
C     IN THIS ILLUSTRATIVE EXAMPLE THE UNKNOWN INITIAL CONDITION, P/PHI,
```

```
C     IS CHANGED BASED UPON THE RESIDUAL IN R.  MANY VARIATIONS ARE
C     POSSIBLE AND THE USER HAS TO DETERMINE WHICH TO USE.  IN THIS CASE
C                    DELYU(1)=RES(1)/DIF(1,6)
C
      DELYU(1)=RES(1)/DIF(1,6)
C
C     THE MAGNITUDES OF THE RESIDUAL VECTOR AND THE VECTOR CHANGE IN UN-
C     KNOWN INITIAL CONDITIONS ARE CALCULATED AND COMPARED TO A PREVIOSLY
C     DEFINED CONVERGENCE CRITERION.
C
      DYUM=DELYU(1)**2
      RESM=0.
      DO 201 I=1,6
  201 RESM=RESM+RES(I)**2
      PRINT 1011
      PRINT(6,2011)ICOUNT
      PRINT ,"HAMILTONIAN(T=0.): ",HB
      PRINT ,"HAMILTONIAN(T=",TOUT,"): ",HA
      PRINT 2030
      PRINT(6,5000)((PHI(I,J),J=1,6),I=1,6)
      PRINT 2030
      PRINT(6,1000)(M(I),I=1,6)
      PRINT 2031
      PRINT(6,1000)(ALPHA(I),I=1,6)
      PRINT(6,3011)(I,RES(I),I=1,6)
      PRINT(6,3010)(I,DELYU(I),I=1,1)
      DO 222 I=1,6
  222 PRINT ,"Y(",I,")= ",Y(I)
      PRINT 1011
      IF (RESM.LT.1.E-14.AND.DYUM.LT.1.E-14) GO TO 250
```

[illegible] QUALITY PRACTICABLE
[illegible] TO DDC

```
      ICOUNT=ICOUNT+1
C
C     THE UNKNOWN INITIAL CONDITIONS ARE CHANGED BY THE AMOUNT DELYU(I).
C
      YU(1)=YU(1)+DELYU(1)
C
C     PARAMETERS ARE RESET TO BEGIN THE INTEGRATION/ITERATION PROCESS
C     AGAIN.
C
      Y(1)=YK(1)
      Y(2)=YK(2)
      Y(3)=YK(3)
      Y(4)=YK(4)
      Y(5)=YK(5)
      Y(6)=YI(1)
      DO 15 J=1,6
      DO 15 K=1,6
      L=6*J+K
   15 Y(L)=IDEN(J,K)
      T=0.
      IFLAG=1
      GO TO 20
  210 CALL OUTPT2(Y,T,YU,YK)
      GO TO 340
  310 PRINT ,"ERROR MESSAGE FROM LEQT1F, IER= ",IER
  340 STOP
      END
      SUBROUTINE INITIAL(RA,EC,KIN,LNG,KI,ROT)
C
C     SUBROUTINE INITIAL CALCULATES THE TWO-BODY ORIT USED AS THE INITIAL
```

```
C     GUESS FOR THE TWO-BODY+J2 AND TWO-BODY+J2+S22,C22 ORBITS.  USING
C     EQUATIONS FROM "FUNDAMENTALS OF ASTRODYNAMICS" BY BATE, MUELLER,
C     AND WHITE, AND WITH
C
C     XI(1)=APOGEE RADIUS, IN DU
C     XI(2)=THETA
C     XI(3)=PHI
C     XI(4)=R/
C     XI(5)=R/THETA
C     XI(6)=R/PHI
C
      DIMENSION XI(6)
      REAL LNG
  100 FORMAT(1X,12HRADIUS(KM): ,F10.2/1X,14HECCENTRICITY: ,F4.2/1X,
     *12HINCLINATION: ,F6.2/1X,11HLONGITUDE: ,F5.2/1X,7HOMEGA: ,E15.3/)
  200 FORMAT(1X,12HRADIUS(TU): ,E16.10/1X,7HTHETA: ,E16.10/1X,5HPHI: ,E1
     *6.10/1X,6HPR/M: ,E16.10/1X,10HPTHETA/M: ,E16.10/1X,8HPPHI/M: ,
     *E16.10/)
      XI(1)=RA/6378.145
      XI(2)=(90.-XIN) 3.141592654/180.
      XI(3)=LNG*3.141592654/180.
      XI(4)=0.
      XI(5)=0.
      A=XI(1)/(1.+EC)
      P=A*(1.-EC**2)
      H=SQRT(P)
      VA=H/XI(1)
      PHPO=VA/(XI(1)*SIN(XI(2)))
      XI(6)=PHPO*XI(1)**2*SIN(XI(2))**2
      PRINT*,"SUBROUTINE INITIAL HAS CALCULATED THE INITIAL CONDITIONS A
```

```
*ACCORDING TO THE INPUT VALUES, APOGEE HEIGHT, ECCENTRICITY, INCLINA
*TION, LONGITUDE, AND OMEGA. THE INPUT VALUES WERE:"
 PRINT(6,100) HA,EC,XIN,LNG,ROT
 PRINT ,"THE INITIAL CONDITIONS ARE:"
 PRINT(6,200)(YI(I),I=1,6)
 RETURN
 END
 SUBROUTINE F(T,Y,YP)
C
C     SUBROUTINE F IS USED BY ODE.  YP(1) THROUGH YP(6) CAN BE RECOG-
C     NIZED AS EQS(12)-(17)(SEE PAGE    ).  THE STATE TRANSITION MATRIX IS
C     ALSO PROPOGATED AT THE SAME TIME.
C
 DIMENSION Y(42),YP(42)
 COMMON/COM1/PHI(6,6),PHIDOT(6,6)
 COMMON/COM2/A(6,6)
 COMMON X1,XK1,XK2,XJ2,S22,C22,OME
 REAL LAM
 COMMON/PA/LAM
 S2=SIN(Y(2))
 C2=COS(Y(2))
 A22=SQRT(C22**2+S22**2)
 C3=COS(2.*Y(3)+LAM)
 S3=SIN(2.*Y(3)+LAM)
 YP(1)=Y(4)
 YP(2)=Y(5)/Y(1)**2
 YP(3)=Y(6)/(Y(1)**2*S2**2)-OME
 YP(4)=Y(5)**2/(Y(1)**3)+Y(6)**2/(Y(1)**3*S2**2)-1./Y(1)
*  **2-1.5*XJ2*(.5+1.5*COS(2.*Y(2)))/Y(1)**4-3.*A22*S3/Y(1)*
-*.
```

```
      YP( )=(Y( )/(Y(1)*S2))+* *C2/S2-XJ2*3.*C2*S2/Y(1)**3
      YP(6)=2.*S22*C3/Y(1)**3
      CALL AMAT(Y,T)
      DO 110 J=1,6
      DO 110 K=1,6
      L=6*J+K
  110 PHI(J,K)=Y(L)
      DO 120 L=1,6
      DO 120 I=1,6
      EL=0.
      DO 12  M=1,6
      EL=EL+A(I,M)*PHI(M,L)
  120 PHIDOT(I,L)=EL
      DO 130 J=1,6
      DO 130 K=1,6
      L=6*J+K
  130 YP(L)=PHIDOT(J,K)
      RETURN
      END
      SUBROUTINE AMAT(Y,T)
C
C     SUBROUTINE AMAT CALCULATES THE ELEMENTS OF THE A MATRIX WHICH IS
C     USED TO PROPOGATE PHI(STATE TRANSITION MATRIX).  THIS IS JUST THE
C     CODED FORM OF THE EQUATIONS SHOWN IN APPENDIX A.
C
      DIMENSION Y( 2)
      COMMON/COM2/A(6,6)
      COMMON XM,XK1,XK2,XJ2,S22,C22,OME
      REAL LAM
      COMMON/P1/LAM
```

```
C2=COS(Y(2))
S2=SIN(Y(2))
A22=SQRT(C22**2+S22**2)
C3=COS(2.*Y(3)+LAM)
S3=SIN(2.*Y(3)+LAM)
A(1,1)=0.
A(1,2)=0.
A(1,3)=0.
A(1,4)=1./XM
A(1,5)=0.
A(1,6)=0.
A(2,1)=-2.*Y(5)/(XM*Y(1)**3)
A(2,2)=0.
A(2,3)=0.
A(2,4)=0.
A(2,5)=1./(XM*Y(1)**2)
A(2,6)=0.
A(3,1)=-2.*Y(6)/(XM*Y(1)**3*S2**2)
A(3,2)=-2.*Y(6)*C2/(XM*Y(1)**2*S2**3)
A(3,3)=0.
A(3,4)=0.
A(3,5)=0.
A(3,6)=1./(XM*(Y(1)*S2)**2)
A(4,1)=-3.*(Y(5)/Y(1))**2/(XM*Y(1)**2)-3.*(Y(6)/(Y(1)*S2))**2/
*(XM*Y(1)**2)+2.*XK1/Y(1)**3+XK2*6.*XJ2*(3.*C2**2-1.)/Y(1)**5+12.
**XK2*A22*C3/Y(1)**5
A(4,2)=-2.*(Y(6)/(Y(1)*S2))**2*C2/(XM*Y(1)*S2)+1.5*XK2*XJ2*(6.*C2
**S2)/Y(1)**4
A(4,3)=-6.*XK2*A22*S3/Y(1)**4
A(4,4)=0.
```

```
      A(4,5)=2.*Y(7)/(XM*Y(1)**3)
      A(4,6)=2.*Y(5)/(XM*(Y(1)*S2)**2*Y(1))
      A(5,1)=-2.*(Y(6)/(Y(1)*S2))**2*C2/(XM*Y(1)*S2)+9.*XK2*XJ2*C2*S2/
     *Y(1)**
      A(5,2)=-(Y(5)/Y(1))**2*(1./S2**2+3.*(C2/S2)**2/S2**2)/XM-3.*XK2*
     *XJ2 (-S2* 2+C2* 2)/Y(1)**3
      A(5,3)=0.
      A(5,4)=0.
      A(5,5)=0.
      A(5,6)=2. Y(6)*C2/(XM*(Y(1)*S2)**2*S2)
      A(6,1)=-5.*XK2*A22*S3/Y(1)**4
      A(6,2)=0.
      A(6,3)=-4.*XK2*A22*S3/Y(1)**3
      A(6,4)=0.
      A(6,5)=0.
      A(6,6)=0.
      RETURN
      END
      SUBROUTINE PRINT(T,Y)
C
C     SUBROUTINE PRINT AND SUBROUTINE PRINT2 ARE JUST PRINTING ROUTINES
C     USED TO DISPLAY SOME OF THE RESULTS.
C
      DIMENSION Y(42)
      COMMON/COM1/PHI(6,6),PHIDOT(6,6)
      COMMON/COM2/A(6,6)
      COMMON XM,XK1,XK2,XJ2,S22,C22,OME
      REAL L1
      COMMON/PA/L1
 2000 FORMAT(//1X,5HTIME: ,E16.10/1X,8HRADIUS: ,E16.10/1X,12HTHETA(DEG)
```

THIS PAGE IS BEST QUALITY PRACTICABLE

```
     *: ,E16.10/1X,10HPHI(DEG): ,E15.10/1X,6HPR/M: ,E16.10/1X,10HPTHETA
     *//: ,E16.10/1X,7HPPHI/M: ,F15.10/1X,4HHM: ,E16.10)
5000 FORMAT(//1X,3HA MATRIX/)
6000 FORMAT(1X,5E20.10)
7000 FORMAT(//1X,23HPHI (STATE TRANSITION MATRIX)/)
8000 FORMAT(//1X,6HPHIDOT/)
     S2=SIN(Y(2))
     C2=COS(Y(2))
     A22=SQRT(C22**2+S22**2)
     C3=COS(2.*Y(3)+LAM)
     S3=SIN(2.*Y(3)+LAM)
     HM=(Y(4)**2+(Y(5)/Y(1))**2+(Y(6)/(Y(1)*S2))**2)*.5/XM-XK1/Y(1)-
     *XK2*XJ2*(.25+.75*COS(2.*Y(2)))/Y(1)**3-XK2*A22*S3/Y(1)**3-Y(6)*OME
     Y2=Y(2)*180./3.141592654
     Y3=Y(3)*180./3.141592654
     PRINT(6,2000)T,Y(1),Y2,Y3,Y(4),Y(5),Y(6),HM
     PRINT 5000
     PRINT(6,6000)((A(I,J),J=1,6),I=1,6)
     PRINT 7000
     PRINT(6,6000)((PHI(I,J),J=1,6),I=1,6)
     PRINT 8000
     PRINT(6,6000)((PHIDOT(I,J),J=1,6),I=1,6)
     RETURN
     END
     SUBROUTINE PRINT2(X,T,YU,YK)
     DIMENSION Y(6),YU(1),YK(6)
     DIMENSION X(6)
     COMMON/COM1/PHI(6,6),PHIDOT(6,6)
     COMMON/COM2/A(6,6)
     REAL LAM
```

```
1010 FORMAT(//1X,32HINITIAL VALUES THAT GAVE PERIODIC ORBIT)
2000 FORMAT(//1X,5HTIME: ,E16.10/1X,8HRADIUS: ,E16.10/1X,12HTHETA(DEG)
    1: ,E16.10/1X,10HPHI(DEG): ,E16.10/1X,6HPR/M: ,E16.10/1X,10HPTHETA
    2/M: ,E16.10/1X,8HPPHI/M: ,E16.10)
5000 FORMAT(//1X,8HA MATRIX/)
6000 FORMAT(1X,6E20.10)
7010 FORMAT(//1X,29HPHI (STATE TRANSITION MATRIX)/)
8000 FORMAT(//1X,6HPHIDOT/)
     Y(1)=YK(1)
     Y(2)=YK(2)
     Y(3)=YK(3)
     Y(4)=YK(4)
     Y(5)=YK(5)
     Y(6)=YJ(1)
     Y2=Y(2)*180./3.141592654
     Y3=Y(3)*180./3.141592654
     PRINT 1010
     T0=0.
     PRINT(6,2000)T0,Y(1),Y2,Y3,Y(4),Y(5),Y(6)
     X2=X(2)*180./3.141592654
     X3=Y(3)*180./3.141592654
     PRINT(6,2000)T,Y(1),X2,X3,X(4),X(5),X(6)
     PRINT 5000
     PRINT(6,6000)((A(I,J),J=1,6),I=1,6)
     PRINT 7010
     PRINT(6,6000)((PHI(I,J),J=1,6),I=1,6)
     PRINT 8000
     PRINT(6,6000)((PHIDOT(I,J),J=1,6),I=1,6)
     RETURN
     END
```

Vita

PII Redacted

James Carlos Garcia was [redacted] He attended the Georgia Institute of Technology from which he received the degree of Bachelor of Aerospace Engineering in June 1977. Upon graduation he received a commission in the U. S. Air Force through the ROTC program. He entered the School of Engineering, Air Force Institute of Technology, in September 1977.

PII Redacted

Permanent Address: [redacted]

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
AFIT/GA/AA/78D-3		

4. TITLE (and Subtitle)
Determination of a 12-Hour Periodic Orbit at the Critical Inclination with the Geopotential Represented through the Second Order Harmonics

5. TYPE OF REPORT & PERIOD COVERED
MS Thesis

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
James C. Garcia
2Lt USAF

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Air Force Institute of Technology (AFIT/EN)
Wright-Patterson AFB, OH 45433

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS

12. REPORT DATE
January 1979

13. NUMBER OF PAGES
41

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release, distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES
APPROVED FOR PUBLIC RELEASE, IAW AFR 190-17
JOSEPH P. HIPPS, MAJOR, USAF
Director of Information

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
The subject of this thesis was an attempt to find a periodic solution to the equations of motion of a high eccentricity (.7) satellite at the critical inclination, with period equal to 12 sidereal hours, and with the earth's potential represented through the second order harmonics. The goals of the study were to (1) find the periodic orbit by numerical means, (2) examine the stability of its motion, and (3) determine the characteristics of the motion near the periodic orbit after including the influences of the sun and moon in the equations of motion. (Cont'd on reverse)

Item 20 cont'd.

Success was obtained in the two-body case for all orbits and in the two-body + J_2 case for a circular equitorial orbit only. In the two-body + J_2 case, no period orbit could be found with non zero inclination and/or eccentricity $\neq 0$. No periodic orbits could be found when S_{22} and C_{22} were included in the equations of motion.

The equations of motion of the orbital elements were examined and used to explain why the periodic orbit could not be found.